Relationships between Pre-service Elementary Teachers' Mathematics Anxiety and Content Knowledge for Teaching

Jim Gleason, Ph.D. †

Abstract

This work builds upon previous research involving individuals' attitude toward mathematics and their achievement in mathematics. We focus on two important components of attitude toward mathematics, namely mathematics test anxiety and numerical anxiety, and their relationship with pre-service elementary teachers' mathematical content knowledge for teaching, which is a distinct construct within achievement in mathematics. The findings of this study are that mathematics test anxiety has a significant correlation with mathematical knowledge for teaching, while the correlation between numerical anxiety and mathematical knowledge for teaching is much lower.

Introduction

Understanding the relationship between the affective and cognitive domains of mathematics education is one important direction toward the improvement of mathematics education. The area of the affective domain that we are concerned about in this study is mathematics anxiety. Mathematics anxiety is "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic settings" (Richardson & Suinn, 1972). Since this mathematics anxiety causes students to have difficulty learning and applying mathematical concepts, a great deal of research appeared in this area following the initial work of Richardson and Suinn (1972) and the publication of the book *Overcoming Math Anxiety* by Sheila Tobias (1978).

The mathematics anxiety of current and prospective elementary teachers and its effect on student learning is one specific area of study related to mathematics anxiety that received a great deal of attention during this period. Some of the research focuses on the extent of mathematics anxiety of prospective elementary teachers. Kelly and Tomhave (1985) found using the MARS instrument that "elementary education majors scored higher on the MARS than any of the other groups except those in the math anxious workshop" (p. 52). On the other hand Becker (1986), using the Fennema-Sherman Mathematics Attitude Scales concluded, "it seems inappropriate to classify this sample of prospective elementary school teachers as having an alarming degree of mathematics anxiety" (p. 51). Other research focused on the transmission of the teacher's mathematics anxiety to the students with similar mixed results (Bulmahn & Young, 1982; Kelly & Tomhave, 1985; Larson, 1983; Martinez, 1987; R. J. Sovchik, 1996; R. Sovchik, Meconi, & Steiner, 1981; Wood, 1988).

Another focus of research surrounding mathematics anxiety, or an individual's attitude toward mathematics, is its relationship with the individual's mathematics content knowledge or achievement in mathematics. Various research studies involving participants in grades K-12 have shown conflicting results with regard to the relationship between mathematics anxiety and mathematics achievement with correlation ranging from below 0.25 to above 0.40 (Ma & Kishor, 1997, p. 27). In their meta-analysis regarding this relationship, Ma and Kishor (1997) found the overall mean effect size was 0.12, which they describe as statistically significant but not strong.

Unfortunately, no such study exists about this relationship among elementary teachers. This is unfortunate since many faculty members consider reducing mathematics anxiety as an important component in the preparation of elementary teachers. This study begins to fill in the gap in the research in order for college faculty members to make informed decisions regarding the goals of mathematics courses for elementary teachers.

Measurement of Mathematics Anxiety

The construct of mathematics anxiety is ambiguous due to a lack of agreement among researchers over its conceptualization (J. Rounds & Hendel, 1980). In practicality, the various instruments designed to measure mathematics anxiety influenced the development of the construct of mathematics anxiety rather than the other direction. The two most prominent instruments currently in use are the Mathematics Anxiety Rating Scale (MARS) developed by Richardson and Suinn (1972), and its many shorter derivations (Alexander & Martray, 1989; Plake & Parker, 1982; J. Rounds & Hendel, 1980; Suinn & Winston, 2003), and the Mathematics Anxiety Scale (MAS) from the Fennema and Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976).

To add to the ambiguity within the construct of mathematics anxiety, several distinct factors have emerged through various factor analytic studies of the most widely used instruments to measure mathematics anxiety. Kazelskis (1998) found that among the RMARS (Alexander & Martray, 1989), the MAS (Fennema & Sherman, 1976), and the MAQ (Wigfield & Meece, 1988) there are six distinct factors. These factors of mathematics test anxiety, numerical anxiety, negative affect toward mathematics, worry, positive affect toward mathematics, and mathematics course anxiety had inter-factor correlations all below 0.500 with the numerical anxiety factor having the only correlation above 0.09 with the mathematics course anxiety factor (Kazelskis, 1998). This is particularly interesting since numerical anxiety fits best with the standard definitions of mathematics anxiety.

The RMARS instrument contained the majority of the items loading in the numerical anxiety factor (Kazelskis, 1998) which corroborates previous studies showing the MARS has the two primary factors of mathematics test anxiety and numerical anxiety (Alexander & Martray, 1989). Since numerical anxiety most closely describes the original definition of mathematics anxiety, our study uses the MARS 30-item (Suinn & Winston, 2003) developed by one of the original authors of the MARS as a shortened version of the MARS while maintaining similar reliability levels and factors as the original.

Measurement of Mathematical Content Knowledge for Teaching

Shulman (1986) first discussed the construct of content knowledge for teaching in regards to any subject area. This content knowledge for teaching that he describes is that “the teacher need not only understand *that* something is so; the teacher must further understand *why* it is so” (Shulman, 1986, p. 9). Additionally, this content knowledge includes an understanding of “why a given topic is particularly central to a discipline whereas another may be somewhat peripheral” (Shulman, 1986, p. 9).

Ball (1991) and colleagues (Ball & Bass, 2000, 2003) further developed this construct in the specific area of mathematics content knowledge for teaching at the elementary school level. This mathematical knowledge for teaching involves “being able to talk about mathematics, not just describing the steps for following an algorithm, but also about the judgments made and the meanings and reasons for certain relationships or procedures” (Ball, 1991).

In an effort to measure this mathematical content knowledge for teaching, the Learning Mathematics for Teaching Project at the University of Michigan developed a series of instruments in the specific areas of number and operation; geometry; and patterns, functions, and algebra (Hill, Schilling, & Ball, 2004; Hill & Ball, 2004; Hill, Rowan, & Ball, 2005). These three sub-constructs within the construct of mathematical knowledge for teaching have been shown as distinct constructs through factor analysis (Hill et al., 2004) and positively predict student gains in mathematical achievement at a level similar to student socio-economic status (Hill et al., 2005). In this study, we use the 2003 version of the number and operation (CKT-M NO), geometry (CKT-M GEO), and patterns, functions, and algebra (CKT-M PFA) as our measurement instruments.

Methodology

Participants; Over a period of two academic semesters, 261 pre-service teachers enrolled in mathematics courses for elementary teachers at a large university in the southeastern United States served as study subjects. The students enrolled in these courses had already completed a traditional mathematics course, usually college algebra, but had not yet completed many courses in education and had limited exposure to the elementary classroom. Since these mathematics courses are prerequisites for many of the education courses involved in the elementary education major, nearly all of the participants were in their freshman or sophomore year at the university.

The participants were 97% female and ranged in age from 19-35 with over 95% being under the age of 22. Additionally, 93% described themselves as Caucasian/White, with 5% African American/Black, and the remaining 2% in other categories.

Instrumentation; Mathematics Anxiety Rating Scale (MARS 30-item):

The MARS 30-item is a shortened version of the original MARS (Richardson & Suinn, 1972) and has a high reliability for internal consistency (Cronback alpha of 0.97) and test-retest reliability (0.90 ($p < 0.001$)) (Suinn & Winston, 2003). In the MARS 30-item, the subjects are asked to state how much they are frightened by each of the 30 statements using 5-point Likert-type responses.

The MARS 30-item retains the two factors of mathematics test anxiety and numerical anxiety from the original MARS by having 15 items that load highly on the mathematics test anxiety factor and 15 items that load primarily onto the numerical anxiety factor (Suinn & Winston, 2003). Some examples of items designed to measure mathematics test anxiety include:

- Thinking about an upcoming math test one hour before.
- Studying for a math test.
- Taking the math section of a college entrance exam.

Some examples of items designed to measure numerical anxiety include:

- Dividing a five digit number by a two digit number in private with pencil and paper.
- Having someone watch you as you total up a column of figures.
- Being given a set of multiplication problems to solve.

Using an exploratory factor analysis, the mathematics test anxiety factor accounted for 59.2% of the variance (eigenvalue=13.02) and the numerical anxiety factor accounted for 11.1% of the variance (eigenvalue=2.44) (Suinn & Winston, 2003). This implies that the MARS 30-item primarily measures mathematics test anxiety while also harvesting some information about an individual's numerical anxiety.

In order to capture the distinctive characteristics of each of these factors, the MARS 30-item instrument was divided into two separate instruments with the mathematics test anxiety instrument (MARS-MTA) consisting of the first 15 items of the MARS 30-item and numerical anxiety instrument (MARS-NA) consisting of the last 15 items. As separate instruments, the MARS-MTA instrument has a reliability of 0.9642 ($p < 0.005$) while the MARS-NA instrument has a reliability of 0.8892 ($p < 0.005$) with the graded model of item response theory (Gleason, 2007). These reliability data imply that by splitting the MARS 30-item into two different instruments, the two new instruments have adequate reliability to give information about groups of individuals, but may not be adequate to measure the mathematical anxiety of individuals.

Content Knowledge for Teaching Mathematics Measure (CKT-M): The Learning Mathematics for Teaching Project at the University of Michigan began in 2001 to develop the Content Knowledge for Teaching Mathematics (CKT-M) series of tests to measure the mathematical knowledge used in teaching elementary mathematics (Hill et al., 2004). These tests consist of multiple-choice questions designed to gain understanding of an individual's knowledge of mathematical content in the three areas of number and operations; geometry; and patterns, functions, and algebra. To have a better idea of the type of items included in these instruments, we include an example of an item in the area of number and operation below.

> Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)
>
> a) Four is an even number, and odd numbers are not divisible by even numbers.
> b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).
> c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.
> d) It only works when the sum of the last two digits is an even number.

The forms used for this study are Form B04 for Elementary Number and Operations (CKT-M NO), Elementary Geometry (CKT-M Geo), and Elementary Patterns, Functions, and Algebra (CKT-M PFA). The CKT-M NO instrument has a reliability of 0.749, the CKT-M Geo has a reliability of 0.861, and the CKT-M PFA has a reliability of 0.733 (Dean, Goffney, & Hill, 2005). While these reliability numbers are not adequate for giving information at the individual level, they are sufficient for attaining information for large group sizes such as the one for this study.

Data Collection; The data was collected from students enrolled in mathematics courses specifically designed for pre-service teachers. The participants completed the survey instruments during a regularly scheduled class time within the first three weeks of classes during two subsequent semesters. They received an adequate amount of time so that all participants were able to complete the instruments within the class period. With this procedure, the sample size was near the recommended sample size of 300 found by Ma and Kishor (1997, p.41) to achieve the best measurement of correlation between attitude toward mathematics and achievement in mathematics.

Results

Following the collection of the data, each participant received scores for the following six categories:

- Mathematics Test Anxiety (MARS-MTA)
- Numerical Anxiety (MARS-NA)
- Number and Operations Content Knowledge (CKT-M NO)
- Geometry Content Knowledge (CKT-M GEO)
- Patterns, Functions, and Algebra Content Knowledge (CKT-M PFA)

The correlations between each of these scores were computed using two-tailed Pearson correlation and are presented in Table I.

As expected, there is a strong correlation (0.441) between mathematics test anxiety and numerical anxiety. Likewise, there are strong correlations between the various types of mathematical knowledge for teaching. However, these correlations are weak enough to imply that these are three distinct constructs within the larger area of mathematical knowledge for teaching, which matches previous findings (Hill et al., 2004).

Mathematics test anxiety correlated with the measured mathematical knowledge for teaching to a much higher degree, approximately 1.5 times as high, as numerical anxiety. This is likely due to mathematical knowledge for teaching being measured by what many of the participants would call a mathematics test. This result is particularly interesting in that mathematics test anxiety is the dominant factor in the MARS instrument and many of its derivations. Therefore, much of the correlations found between mathematics anxiety and achievement in mathematics may be influenced by an individuals test anxiety rather than mathematics anxiety.

An individual's numerical anxiety does correlate with their mathematical knowledge for teaching. These correlations are on the lower scale of the ranges found by Ma and Kishor (1997) in the relationship between attitude toward mathematics and achievement in mathematics. In particular, with correlations below 0.20, this relationship is weak and may not have practical significance in the behavioral sciences (Cohen & Cohen, 1983; Rosenthal & Rubin, 1982).

Conclusions

These results that mathematics test anxiety is significantly correlated with mathematical knowledge for teaching while numerical anxiety has a weak correlation implies that when reporting results involving mathematics anxiety, one should specifically identify the construct within mathematics anxiety that is being measured. This is particularly important for studies involving pre-service teachers, in that faculty members need to know what construct within mathematics anxiety they need to address to improve the future teachers' ability to generate student success.

Since the correlation between mathematics test anxiety and mathematical knowledge for teaching is significant, this might be one controlled variable in studies involving mathematical knowledge for teaching. For example, the relationship between a teacher's mathematical knowledge for teacher and the students' knowledge growth might be stronger when the researcher accounts for the effect of the teacher's mathematics test anxiety when measuring his or her mathematical knowledge for teaching.

Finally, since the relationship between an individual's numerical anxiety and mathematical knowledge for teaching is weak and that his or her mathematical knowledge for teaching has a stronger implication toward student achievement, perhaps we should focus more on improving an individual's mathematical knowledge for teaching and less on mathematics anxiety and other dispositions toward mathematics. However, before such a conclusion can be made, there must be further research in this area.

Table I
Correlation Table

	MARS-NA	CKT-M NO	CKT-M GEO	CKT-M PFA
MARS-MTA	0.441**	-0.254**	-0.246**	-0.173**
Sig. (2-tailed)	0.000	0.000	0.000	0.005
MARS-NA		-0.170**	-0.162**	-0.137*
Sig. (2-tailed)		0.006	0.009	0.027
CKT-M NO			0.292**	0.231**
Sig. (2-tailed)			0.000	0.000
CKT-M GEO				0.462**
Sig. (2-tailed)				0.000

N=261
** Correlation is significant at the 0.01 level (2-tailed)
* Correlation is significant at the 0.05 level (2-tailed)

† *Jim Gleason*, Ph.D, The University of Alabama, Alabama, USA

References

Alexander, L., & Martray, C. (1989). The development of an abbreviated version of the Mathematics Anxiety Rating Scale. *Measurement and Evaluation in Counseling and Development*, 22, 143-150.

Ball, D. L. (1991). Teaching mathematics for understanding: What do teachers need to know about subject matter? In M. Kennedy (Ed.), *Teaching academic subjects to divers learners* (p. 63-83). New York: Teachers College Press.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple Perspectives on the Teaching and Learning of Mathematics* (p. 83-104). Westport, CT: Ablex.

Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis & E. Simmt (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group* (p. 3-14). Edmonton, AB: CMESG/GCEDM.

Becker, J. R. (1986). Mathematics attitudes of elementary education majors. *Arithmetic Teacher*, 33 (5), 50-51.

Bulmahn, B. J., & Young, D. M. (1982). On the transmission of mathematics anxiety. *Arithmetic Teacher*, 30 (3), 55-56.

Cohen, J., & Cohen, P. (1983). *Applied multiple regression/correlation analysis in behavioral sciences*. Hillsdale, NJ: Erlbaum.

Dean, C. A., Goffney, I. M., & Hill, H. C. (2005). *Technical report on the validity of CKT-M items* (Tech. Rep.). Ann Arbor, Michigan: University of Michigan.

Fennema, E., & Sherman, J. (1976). Fennema-Sherman mathematics attitude scales: Instruments designed to measure attitudes toward learning mathematics by females and males. *JSAS Catalog of Selected Documents in Psychology*, 6, 31.

Gleason, J. (2007). Using the mathematics anxiety rating scale to measure the mathematical dispositions of pre-service elementary teachers. (submitted)

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes. *Journal for Research in Mathematics Education*, 35 (5), 330-351.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on students' achievement. *American Educational Research Journal*, 42 (2), 371-406.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing Measures of Teachers' Mathematics Knowledge for Teaching. *The Elementary School Journal*, 105 (1), 11-30.

J. Rounds, J., & Hendel, D. (1980). Measurement and dimensionality of mathematics anxiety. *Journal of Counseling Psychology*, 27, 138-149.

Kazelskis, R. (1998). Some dimensions of mathematics anxiety: a factor analysis across instruments. *Educational and Psychological Measurement*, 58, 623-633.

Kelly, W. P., & Tomhave, W. K. (1985). A study of math anxiety/math avoidance in preservice elementary teachers. *Arithmetic Teacher*, 32 (5), 51-53.

Larson, C. N. (1983). Techniques for developing positive attitudes in preservice teachers. *Arithmetic Teacher*, 31 (2), 8-9.

Ma, X., & Kishor, N. (1997). Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analysis. *Journal for Research in Mathematics Education*, 28 (1), 26-47.

Martinez, J. G. R. (1987). Preventing math anxiety: A prescription. *Academic Therapy*, 23, 117-125.

Plake, B. S., & Parker, C. S. (1982). The development and validation of a revised version of the Mathematics Anxiety Rating Scale. *Educational and Psychological Measurement*, 42 (2), 551-557.

Richardson, F., & Suinn, R. (1972). The Mathematics Anxiety Rating Scale: Psychometric data. *Journal of Counseling Psychology*, 19, 551-554.

Rosenthal, R., & Rubin, D. B. (1982). A note on percent variance explained as a measure of the importance of effects. *Journal of Applied Social Psychology*, 9, 395-396.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15 (2), 4-14.

Sovchik, R., Meconi, L. J., & Steiner, E. (1981). Mathematics anxiety of preservice elementary mathematics methods students. *School Science and Mathematics*, 81 (8), 643-648.

Sovchik, R. J. (1996). *Teaching mathematics to children*. New York: Harper Collins.

Suinn, R. M., & Winston, E. H. (2003). The Mathematics Anxiety Rating Scale, a brief version: Psychometric data. *Psychological Reports*, 92, 167-173.

Tobias, S. (1978). *Overcoming Math Anxiety*. New York: W.W. Norton and Company, Inc.

Wigfield, A., & Meece, J. (1988). Math anxiety in elementary and secondary school students. *Journal of Educational Psychology*, 80, 210-216.

Wood, E. F. (1988). Math anxiety and elementary teachers: What does research tell us? *For the Learning of Mathematics*, 8 (1), 8-13.